Baryon Self-Energy With QQQ Bethe-Salpeter Dynamics In The Non-Perturbative QCD Regime: n-p Mass Difference

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Abstract

A *qqq* BSE formalism based on $DB\chi S$ of an input 4-fermion Lagrangian of 'current' u, d quarks interacting pairwise via gluon-exchange- propagator in its *non-perturbative* regime, is employed for the calculation of baryon self-energy via quark-loop integrals. To that end the baryon-*qqq* vertex function is derived under Covariant Instantaneity Ansatz (CIA), using Green's function techniques. This is a 3-body extension of an earlier $q\bar{q}$ (2-body) result on the exact 3D-4D interconnection for the respective BS wave functions under 3D kernel support, precalibrated to both $q\bar{q}$ and qqq spectra plus other observables. The quark loop integrals for the neutron (n) - proton (p) mass difference receive contributions from : i) the strong SU(2) effect arising from the $d - u$ mass difference (4 MeV); ii) the e.m. effect of the respective quark charges. The resultant $n - p$ difference comes dominantly from $d - u$ effect ($+1.71 Mev$), which is mildly offset by e.m.effect (-0.44), subject to gauge corrections. To that end, a general method for QED gauge corrections to an arbitrary momentum dependent vertex function is outlined, and on on a proportionate basis from the (two-body) kaon case, the net n-p difference works out at just above 1 MeV. A critical comparison is given with QCD sum rules results.
PACS : 11.10 st ; 12.35 Ht ; 12.70 + q Key-words: baryon self-energy; qqq BS-vertex fn; Cov.Inst.Ansatz; 3D-4D interconnection; Green's fn method; QED gauge inv; n-p diff.

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1 Introduction:Relativistic 2- and 3-Quark Hadrons

Soon after the advent of the Faddeev theory [1], the *relativistic* 3-body problem [2] attracted instant attention as a non-trivial *dynamical* problem, as distinct from earlier "kinematical" attempts [3] at a relativistic formulation of its wave function. In this respect the relativistic 3-baryon problem had been more of academic than practical interest (until the 'pion' got involved as a key ingredient), but the situation changed qualitatively when this 3-body problem started being viewed at the quark level. Looking back after 25 years it appears that the first serious attempt in this direction was made by Feynman et al [4] who gave a unified formulation of both the $q\bar{q}$ (meson) and qqq (baryon) problems under a common dynamical framework, bringing out rather sharply an underlying duality between these two systems which in turn signifies a more basic duality between a qq diquark [5] and a $\bar{q}$ antiquark. Indeed the diquark description is quite compact and adequate for many practical purposes involving the baryon, but the more microscopic qqq description which brings out the fuller permutation (S_3) symmetry in the baryon is necessary for the actual details of a full-fledged dynamical treatment [4].

1.1 BS Dynamics:3D vs 4D Forms

Although the FKR theory [4] marked the first step in this direction, it suffered from an inadequate treatment of the time-like d.o.f. which showed up in several ways. The latter has by itself a long history of attempts at 3D formulations of the 4D Bethe-Salpeter Equation (BSE) for $q\bar{q}$ qqq systems:Instantaneous Approximation (IA) [6a]; Quasipotentials [6b]; On-shellness of the associated propagators [6c]; and others [6d]. Some of these approaches have been reviewed elsewhere [7] in the context of a unified BS formulation of $q\bar{q}$ and qqq in the FKR [4] spirit.

A different form of 3D reduction, which is of more recent origin [8,9] is based on the Markov-Yukawa Transversality Condition [10] in which the BSE kernel is given a 3D support by demanding that it be a function of the relative momentum $\hat{q}$ transverse to the total 4-momentum P. This condition, termed the "Covariant Instaneity Ansatz" (CIA) [8], is somewhat complementary to the more conventional approaches [6] in which the BS kernel is left untouched but the propagators are manipulated in various ways to give a 3D reduction to the BSE, whereas in the Markov-Yukawa [8-10] method, which has been termed CIA [8], it is the other way around. In the conventional 3D reductions [6] there is no going back to the original 4D form, whereas in the CIA [8], the two forms are fully interchangeable according to need, thus offering a possible Lorentz covariant way to reconcile the apparently conflicting demands of spectroscopy [11] needing a 3D BSE, with the 4D BS vertex functions needed for quark-loop integrals. The effectiveness of the CIA in giving a concrete shape to such a "two-tier" philosophy of spectra-cum-loop integrals was summarised in a semi-review [12] in the form of appropriate BSE's for both $q\bar{q}$ and qqq systems with vector-type kernels [7] with 3D support, albeit with slight modifications [7] in the respective BSE structures to facilitate greater 'manoeuvreability', in the spirit of similar efforts [13] in the past. Further, the observed spectroscopy [11] is well satisfied on both $q\bar{q}$ [14] and qqq [15] sectors with a *common set* of parameters for the respective kernels (the qq kernel has just half the strength of the $q\bar{q}$ kernel due to color effects), so that the respective vertex functions are entirely determined within the CIA formalism.

1.2 QCD-Motivated BSE in 3D-4D Form

The other aspect of this 'two-tier' formalism concerns the crucial property of chiral symmetry *and* its dynamical breaking. The first part (chiral symmetry) is ensured without extra charge by the *vector* character of the kernel that had been present all along in this program [7,16], since the BS-kernel is a direct reflection of an effective 4-fermion term in the input Lagrangian. Indeed the vector type character of the latter lends a natural gluon exchange flavour to such a pairwise interaction among 'current' (almost massless) u, d quarks at the Lagrangian level. This structure is quite general [17], and can be adapted to the QCD requirements on the gluonic propagator involved in the pairwise interaction kernel. Of this, the perturbative part (which is well understood) is quite explicit, but the *non-perturbative* (infrared) part is not yet derivable from formal QCD premises [18]. It can nevertheless be simulated in a sufficiently realistic manner at the phenomenological level [7,19], so as to satisfy the standard constraints of confinement as well as explicit QCD features [20] in terms of a basically 3D BSE kernel structure.

The second part, viz., dynamical breaking of chiral symmetry ($DB\chi S$) is implemented via the Nambu-Jonalasino mechanism [21] whose full-fledged form amounts to adopting the 'non-trivial' solution of the Schwinger-Dyson Equation (SDE) derived from a given input, chirally symmetric Lagrangian with current quarks. A mass function $m(p)$ [17,18] is thus generated whose low-momentum value may be identified with the bulk of the 'constituent' mass (m_q) of the u, d quarks. This accords with Politzer additivity [22], viz., $m_q = m(0) + m_c$; where m_c, the current mass, is small. This was also shown in the context of a BSE-cum-SDE treatment [23] within the CIA formalism [8]. Thus formally the BS-kernel may be regarded as a *non-perturbative* gluon propagator [23] in a BSE framework involving the dynamical/constituent mass [19, 21-24] in the quark propagator.

To recapitulate, the CIA which gives an exact interconnection between the 3D and 4D forms of the BSE, provides a unified view of 2- and 3-quark hadrons, its 3D reduction being meant for spectroscopy [14-15], and the reconstructed 4D form [12,13] for identifying the respective hadron-quark vertex functions as the key ingredients for 4D quark-loop integrals. The formalism stems from a strongly QCD-motivated Lagrangian with *current* quarks whose pairwise interaction is mediated by a gluonic propagator in its non-perturbative regime. The QCD feature of chiral symmetry is ensured by the vector nature of this interaction, while its dynamical breaking is the result of a non-trivial solution of the SDE [17,23]. Thus, unlike in conventional potential models [25], the constituent mass so generated is *not* a phenomenological artefact, but the result of a self-consistent solution of the SDE [17, 19, 23], so that the standard (constituent) mass employed for spectroscopy [14-15] 'checks' with the output dynamical mass at low momentum [23]. Thus there are only two genuine input parameters C_0, ω_0, that characterize the (phenomenological) structure of the non-perturbative gluon propagator which serves for both the 2- and 3-quark spectra in a unified fashion [14-15]. In this formalism, these two constants play a role somewhat similar to that of the (input) 'condensates' in the theory of QCD sum rules [26].

1.3 Comparison With Chiral Perturbation Theory, Etc

Before proceeding further, let us pause to compare this approach with other dynamical methods, e.g., chiral perturbation theory [27] which has more explicit QCD features, albeit

in the perturbative regime, leading to expansions in the momenta. This is a powerful theoretical approach employing the (chiral) symmetry of QCD; its essential parameters are the current quark masses, and the method works very efficiently where its premises are logically applicable. Thus it predicts the *ground state* spectra of light quark hadrons, including their mass splittings due to strong and e.m. breaking of SU(2), but *not* the spectra of L-excited hadrons. The latter on the other hand demand a "closed form" approach to incorporate the "soft" *off-shell* effects which in turn require a non-trivial handle on the *infrared* (non-perturbative) part of the gluonic propagator, something which the present state of the QCD art does not yet provide. Thus one needs a phenomenological input even in standard BSE-SDE approaches [18], as discussed elsewhere [23]. The chiral perturbation theory [27] also lacks this vital ingredient, as seen from the absence of form factors in its 'point' Lagrangians [27] with at most derivative terms. This shows up, e.g., through its inability to predict L-excited spectra, and finer aspects (such as convergence) of 4D quark-loop integrals which depend crucially on these "off-shell" features. *Physically* this amounts to the absence of a 'confinement scale' which governs these form factors. In other BSE-cum-SDE approaches [17-19], including the present 'two-tier' CIA formalism [8,12], this 'scale' is an integral part of the structure of the non-perturbative part of the gluon propagator [19,23], with a built-in QCD feature of chiral symmetry and its dynamical breaking through the non-trivial solution of the SDE [17,19,23]. This not only facilitates the prediction of L-excited spectra [19,14-15] but also provides a form factor for the hadron-quark vertex function which greatly enhances its applicability to various 4D quark-loop integrals; see [8, 23-24, 28].

1.4   Application to n-p Mass Difference with 3D-4D qqq BSE

After this excursion on the philosophy of this two-tier BSE approach, vis-a-vis some others [26,27], we may now state the objective of the present paper: A typical application of the 4D baryon-qqq wave function reconstructed [12] from the 3D qqq BSE, to the $n - p$ mass difference, as a 3-body generalization of the corresponding $q\bar{q}$-meson problem [28]. Unlike the 2-body case, however, where the 3D-4D interconnection is exactly reversible [8], a 4D reconstruction for a 3-body system involves a loss of information on the 4D Hilbert space, so that the reversal of steps is in principle non unique, and requires a 1D δ-function to fill up the information gap between 3D and 4D Hilbert space which may be directly attributed to the CIA ansatz of a 3D support to the pairwise kernel. The 2-body case just escapes this pathology as it represents a sort of degenerate situation, but the price of a 3D kernel support must show up in a reconstruction of the 4D BSE from its reduced 3D form in any $(n > 2)$-body problem [29]. A plausible 'CIA' structure for the 4D qqq wave function was suggested in [12] in a semi-intuitive fashion, but a more formal mathematical basis has since been found [29] through the use of Green's function techniques, so that the reconstructed 4D form reduces exactly to the (known) 3D form as a consistency check [29]. The final result, which is almost the same as the earlier conjecture, eq.(5.15) of [12], except for a constant that does not affect the normalization, contains a 1D δ-function corresponding to the on-shell propagation of the spectator between two successive vertex points. As explained in detail in [29], this 1D δ- function must not be confused with any signature of "non-connectedness" in the 3-body wave function [30], since the 3D form is *fully connected*. Rather, it can be likened to a (Fermi-type) δ-function potential in estimating the effect of chemical binding on the scattering of very slow neutrons by a

hydrogen molecule [31], as a practical devise to fill up the vast mismatch of scales in the interactions at nuclear vs molecular levels. In any case the 1D δ-function appearing in this structure is entirely innocuous as it gets integrated out in any *physical* (quark loop) amplitude including the BS normalization (see Sec.2 below).

Now to recall the *physics* of the n-p mass difference, this quantity receives contributions of *opposite* signs from two main sources: i) a positive one from the strong SU(2) $d-u$ mass difference; ii) a negative one from e.m. splittings. A third source, the effect of quark condensates, which plays a crucial role in QCD sum rule studies [26] gives rather small contributions in this non-perturbative approach, as already found in the meson case [28], and will therefore be neglected; for a detailed discussion on this issue, see sec.5.2.

On the other hand the problem of gauge invariance (g.i.) of the e.m. contribution (ii) is a more tricky issue in view of the otherwise arbitrary nature of the extended form factor associated with the *baryon* $-qqq$ vertex function, in contrast with the *point-like* vertices involved, e.g., in the corresponding QCD-sum rule studies [32]. For the same reason, the g.i. issue could not be addressed in [28] for the meson case, in the hope that it would be of the same order of magnitude as the relatively small (20of the e.m. contribution [28]. The g.i. problem with arbitrary hadron-quark vertex functions is best left to a separate, more substantial investigation. In the meantime in this paper we shall estimate the g.i. corrections for a *2-body* problem on the lines of [32] adapted to an arbitrary meson- quark vertex function. The steps are indicated in an appendix (Appendix C) for the kaon problem as a test case, the results of which are provisionally considered as an indication of the nature of the g.i. corrections to be expected for the 3-body problem on hand, pending a formal treatment later. The kaon result indicates an increase of 0.612 MeV in the e.m. self-energy (1.032 MeV) arising from fig 1(b) of [28].

1.5 Contents of the Paper

In Sec.2 we collect the various pieces of the central quantity of the present investigation, viz., the 4D $baryon-qqq$ vertex function in terms of 3D quantities, with the inclusion of the spin and isospin d.o.f. on the lines of an earlier study [33]. Thus equipped, we outline the main steps leading to an explicit evaluation of the normalization integral, using Feynman diagrams shown in figs.1(a.b.c). A *complex* basis [9, 34, 35] for 3D momentum variables facilitates the evaluation of the resulting $3D \times 3D$ integrals, *after* the time-like momenta have been eliminated by 'pole' integrations on identical lines to the corresponding $q\bar{q}$ problem [12,23,24]. In Sec.3 we evaluate the 'shift' in the nucleon mass due to strong SU(2) breaking, by inserting a mass shift operator $-\delta m\tau_3{}^{(i)}/2$ in place of $i\hat{\gamma}_\mu e_i$ at each of the corresponding γ- vertices of figs.1(a,b,c), as shown in figs. 2.(a,b,c). Here $\delta m = 4$ MeV is the 'standard' d-u mass difference [24,28] taken as the basic input. Sec.4 sketches the evaluation of the e.m. contribution in accordance with the diagrams of fig.3(a,b,c). The details of the e.m. approximations employed are collected in Appendix A. Appendix B sketches the main steps of the derivation [29] for the 4D structure, eq.(5.15) of [12], of the baryon-qqq vertex function by the Green's function method for 3 spinless quarks. Finally Appendix C sketckes the steps leading to the g.i. corrections [32] for the kaon case, as a sort of facsimile of similar corrections expected for the $n-p$ problem on hand. Sec.5 summarises our findings and conclusions vis-a-vis other methods.

2   Normalization of the Baryon-qqq Vertex Function

To outline the structure of the baryon-qqq vertex function from a CIA-governed BSE [12-13], we shall generally follow the notation, normalization and phase convention for the various symbols as given in [13], but adapted to the *equal* mass kinematics ($m_1 = m_2 = m_3 = m_q$). The SU(2) mass difference δm ($\approx 4 MeV$) between d and u quarks will be taken into account only through a 2-point vertex $[-\delta m \tau_3{}^{(i)}/2]$ inserted in the quark propagators in figs.2 (in place of $i\gamma_\mu e_i$ for a photon), but not in the structure of the vertex function. The vertex function is written in *three* pieces in each of which *one* quark plays the role of the 'spectator' by turn. For the spin structure (not given in [13]) we employ the convention of [3] which was extended in [33] to incorporate the S_3-symmetry for the spin-cum- isospin structure in the Verde [36] notation [37]. The full 4D BS wave function Ψ reads as [13,33,34] :

$$\Psi \Delta_1 \Delta_2 \Delta_3 = (\Gamma_1 + \Gamma_2 + \Gamma_3) \times [\chi'\phi' + \chi"\phi"]/\sqrt{2}; \tag{2.1}$$

$$\Delta_i = m_q{}^2 + p_i{}^2; \quad (i = 1, 2, 3). \tag{2.2}$$

Here χ' and $\chi"$ are the relativistic "spin" wave functions in a 2-component mixed symmetric S_3 basis which for a **56** baryon go with the associated isospin functions ϕ' and $\phi"$ respectively. These are given by [3,33] :

$$[\chi']_{\beta\gamma;\alpha} = [(M - i\gamma.P)i\gamma_5 C/\sqrt{2}]_{\beta\gamma} \times U(P)_\alpha/(2M) \tag{2.3}$$

$$[\chi"]_{\beta\gamma;\alpha} = [(M - i\gamma.P)\gamma_\mu C/\sqrt{6}]_{\beta\gamma} \times i\gamma_5\gamma_\mu U(P)_\alpha/(2M) \tag{2.4}$$

in a spinorial basis [3,33] in which the index α refers to the 'active' quark (interacting with an external photon line, fig.1), while β, γ characterize the other two, with the further convention that γ refers to the "spectator" in a given diagram, fig.(1). The 'hat' on γ signifies its perpendicularity to P_μ, viz., $\hat{\gamma}.P = 0$. The notations in eqs.(2.3-4) are standard, with a common Dirac basis for the entire structure, and 'C' is the charge conjugation operator for quark #3 in a 23-grouping [3,33]. P_μ is the baryon 4-momentum, $U(P)$ is its spinor representation, and $(M - i\gamma.P)/(2M)$ its energy projection operator [3,33]. Further, because of the full S_3-symmetry of the last factor in (2.1), the $(1, 2, 3)$ indices can be permuted as needed for the diagram on hand. Thus in fig.1a, #1(α) interacts with the photon ; #2(β) is the quark which has had a 'last' qq-interaction with #1(α) before emerging from the hadronic 'blob', while #3(γ) is the spectator [33]. In fig.1b, the roles of #1 and #2 are reversed so that, of the two 'active quarks' #1 and #2, #2(α) now interacts with the photon, #1(β) has had the last qq- interaction with #2(α), while #3(γ) still remains the 'spectator'. These roles are cyclically permuted, with two more such pairs of diagrams, fig.1c), to give an identical chance to each of the quarks in turn [33]. Thus there are 3 such pairs of diagrams, of which only one pair is shown. An identical consideration applies to figs.2(a,b) with $i\gamma_\mu e_i$ replaced by $(-\delta m \tau_3/2)$ consistently. The spatial vertex functions Γ_i are given for i = 3 by [12] :

$$\Gamma_3 = N_B[D_{12}\phi/2i\pi] \times \sqrt{[2\pi\delta(\Delta_3).\Delta_3]} \tag{2.5}$$

where ϕ is the full, *connected* qqq wave function in 3D form, and D_{12} is the 3D denominator function of the (12) subsystem . The second factor represents the effect of the spectator

[12] whose *inverse* propagator $D_F{}^{-1}(p_3)$ *off the mass shell* is just Δ_3, eq.(2.2). The main steps leading to this unorthodox structure which has been derived recently via the techniques of Green's functions [29], are sketched for completeness in Appendix B. As already noted in Sec.1, and again explained in Appendix B, its peculiar singularity structure in the form of a "square-root" of a 1D δ-function stems from the CIA ansatz of a 3D support to the pairwise interaction kernel, but it is quite harmless as the former will appear in a *linear* form in the transition amplitude corresponding to any Feynman diagram as in figs.1-2. The complete expressions for D_{12} and ϕ are given for the equal mass case (with #3 as spectator) by (see [12,15]):

$$D_{12} = \Delta_{12}(M - \omega_3); \quad \Delta_{12} = 2\omega_{12}{}^2 - M^2(1 - \nu_3)^2/2 \tag{2.6}$$

$$\omega_{12}{}^2 = m_q{}^2 + \hat{q}_{12}^2; \quad 2\hat{q}_{12}^\mu = \hat{p}_1^\mu - \hat{p}_2^\mu \tag{2.7}$$

$$\phi = e^{-(\hat{p}_1^2 + \hat{p}_2^2 + \hat{p}_3^2)/2\beta^2} \equiv e^{-\rho/3\beta^2} \tag{2.8}$$

(see further below for the definition of ρ).

$$\hat{p}_i^\mu = p_i{}^\mu + p_i.PP_\mu/M^2; \quad \hat{p}_1^\mu + \hat{p}_2^\mu + \hat{p}_3^\mu = 0 \tag{2.9}$$

$$\omega_i{}^2 = m_q{}^2 + \hat{p}_i^2; \quad \nu_3 = \omega_3/M(onshell) \tag{2.10}$$

The β-parameter is defined sequentially by [14,15]:

$$\beta^4 = \frac{4}{9} M\omega_0{}^2 \bar{\alpha}_s (1 - m_q/M)^2 (M - <\omega>); \quad <\omega>^2 = m_q{}^2 + 3\beta^2/8 \tag{2.11}$$

$$\bar{\alpha}_s{}^{-1} = \alpha_s{}^{-1} - 2MC_0 \frac{(1 - m_q/M)^2}{M - <\omega>}; \tag{2.12}$$

$$\frac{6\pi}{\alpha_s} = 29\ln\frac{(M - <\omega>)}{\Lambda_{QCD}}; \tag{2.13}$$

$$\Lambda_{QCD} = 200 MeV; \quad \omega_0 = 158 MeV; \quad C_0 = 0.29 \tag{2.14}$$

The normalization N_B, eq.(2.5), is given in accordance with the Feynman diagrams 1(a,b) by the 4D integral (c.f.[33]) :

$$iP_\mu/M = \sum_{123} \int d^4q_{12} d^4p_3 \frac{\Gamma_3{}^*\Gamma_3}{2\Delta_2\Delta_3} [<\phi'|(23)'(1)'_\mu|\phi'> + \frac{1}{3} <\phi''|(23)''_{\nu\lambda}(1)''_{\nu\lambda;\mu}|\phi''>]$$
$$+(1 \Leftrightarrow 2) \tag{2.15}$$

where the matrix element for fig.1a is organized as a product of two spin-factors : a '23-element' expressed as a Dirac trace over the indices β, γ; and a '1-element' (with suppressed index α). The associated isospin functions ϕ are shown according to (2.1). The contribution of fig.1b is shown symbolically by $1 \Leftrightarrow 2$, while $\sum_{123}$ indicates the sum over all the 3 pairs cyclically. In representing eq.(2.12) we have dropped 'cross-terms' like $\Gamma_i{}^*\Gamma_j$, where $i \neq j$, since the presence of a $\sqrt{\delta}$-function in each Γ_i ensures that a simultaneous 'on-shell' energy conservation of $i \neq j$ spectators is not possible [33]. The various pieces of the matrix elements in (2.14) which can be read off from fig.1a in terms of the spin functions (2.3-4) are as follows:

$$(1)'_{\nu\lambda;\mu} = \bar{U}(P)S_F(p_1)i\gamma_\mu e_1 S_F(p_1)U(P) \tag{2.16}$$

$$iS_F{}^{-1}(p) = m_q + i\gamma.p \tag{2.17}$$

$$(1)"_{\nu\lambda;\mu} = \bar{U}(P)i\hat{\gamma}_\nu\gamma_5 S_F(p_1)i\gamma_\mu e_1 S_F(p_1)i\gamma_5\hat{\gamma}_\lambda U(P); \tag{2.18}$$

$$(23)' = Tr[C^{-1}\gamma_5(M - i\gamma.P)(m_q - i\gamma.p_2)(M - i\gamma.P)\gamma_5(m_q + i\gamma.p_3)C]/8M^2 \tag{2.19}$$

$$(23)"_{\nu\lambda} = Tr[C^{-1}\hat{\gamma}_\nu(M - i\gamma.P)(m_q - i\gamma.p_2)(M - i\gamma.P)\hat{\gamma}_\lambda(m_q + i\gamma.p_3)C]/8M^2 \tag{2.20}$$

The 'strength' e_i of the (zero-momentum) 'photon' coupling to the quark line p_i can be chosen in several ways [38]. We take here the simplest possibility, viz., $e_i = 1/3$ each. The isospin matrix element is first eliminated according to [39]:

$$< \phi'|1|\phi' >=< \phi"|1|\phi" >= 1 \tag{2.21}$$

$$< \phi'|\tau_3{}^{(1)}|\phi' >= -3 < \phi"\tau_3{}^{(1)}|\phi" >=< \tau_3 >_{(p,n)} \tag{2.22}$$

Eq.(2.20) suffices for (2.4), while (2.21) will be needed for the u-d mass difference operator $-\delta m\tau_3{}^{(1)}/2$; see Sec.3. Next, the evaluation of the traces in (2.15-18) is straightforward, after noting that (2.15-16), *after* spin-averaging, are expressible as traces. The results are

$$(23)'\theta_{\nu\lambda} = (23)"_{\nu\lambda} = (m_q + M\nu_2)(m_q + M\nu_3)\theta_{\nu\lambda} \tag{2.23}$$

$$(1)'_\mu\theta_{\nu\lambda} = (1)"_{\nu\lambda;\mu} = [2M\nu_1(m_q + M\nu_1) + \Delta_1]\theta_{\nu\lambda}P_\mu/(M\Delta_1{}^2) \tag{2.24}$$

where θ is a covariant Kronecker delta w.r.t. P_μ, viz.,

$$\theta_{\nu\lambda} \equiv \theta_{\nu\lambda} = \delta_{\nu\lambda} - P_\nu P_\lambda/P^2; \quad (P^2 = -M^2) \tag{2.25}$$

Collecting all these results and simplifying we get

$$N_B{}^{-2} = \sum_{123}\int d^3\hat{p}_3\frac{(m_q + \omega_3)}{2\omega_3} \times \int d^3\hat{q}_{12}D_{12}{}^2\phi^2[e_1 I_1 + e_2 I_2] \tag{2.26}$$

$$2i\pi I_1 = \int Md\sigma_{12}[2M\nu_1(m_q + M\nu_1) + \Delta_1]/(M\Delta_1{}^2\Delta_2) \tag{2.27}$$

where we have "cashed" the $\delta(\Delta_3)$-function arising from $|\Gamma_3|^2$ against the time-like component of d^4p_3, and used the results

$$d^4q_{12} = d^3\hat{q}_{12}Md\sigma_{12}; \quad \nu_{1,2} = (1 - \nu_3) \pm \sigma_{12} \tag{2.28}$$

The integration over $d\sigma_{12}$ involves single and double poles arising from the propagators $\Delta_{1,2}{}^{-1}$ in (2.26), while the value of ν_3 is taken 'on-shell' at ω_3/M after the $\delta(\Delta_3)$- function has been cashed. The result of a basic σ_{12}-integration is

$$\int Md\sigma_{12}\Delta_1{}^{-1}\Delta_2{}^{-1} = 2i\pi/D_{12} \tag{2.29}$$

from which others can be deduced by differentiation under *unequal* mass kinematics, or directly through a 'double pole' integration. The net result for $I_1 + I_2$, eq.(2.26), is given in eq.(2.41) below. Further, the individual terms of the summation $\sum_{123}$ in (2.25) are fixed by the values chosen for e_i (which need not be specified in advance, as they can be adapted to other conventions too [38]; see Sec.3).

The integration in (2.25) can be considerably simplified in a complex basis [15,34] defined (in momentum space) by :

$$\sqrt{2}z_i = \xi_i + i\eta_i; \quad \sqrt{2}z_i{}^* = \xi_i - i\eta_i; \tag{2.30}$$

$$\sqrt{3}\xi_i = p_{1i} - p_{2i}; \quad 3\eta_i = -2p_{3i} + p_{1i} + p_{2i}; \tag{2.31}$$

where we now employ the alternative notation p_{1i} for $\hat{p}_1^\mu$, in view of its basically 3D content. In terms of z_i and $z_i{}^*$, the 6D integration in (2.25) is expressed as

$$d^3\hat{p}_3 d^3\hat{q}_{12} = (\sqrt{3}/2)^3 d^3\xi d^3\eta = d^3z d^3z^* \tag{2.32}$$

The further representation [15,34]

$$d^3z d^3z^* = (dz_+ dz_-{}^*)(dz_- dz_+{}^*)(dz_3 dz_3{}^*) \tag{2.33}$$

where

$$\sqrt{2}z_+ = R_1 \, e^{i\theta_1}; \quad \sqrt{2}z_-{}^* = R_1 \, e^{-i\theta_1} \tag{2.34}$$

$$\sqrt{2}z_- = R_2 \, e^{i\theta_2}; \quad \sqrt{2}z_+{}^* = R_2 \, e^{-i\theta_2} \tag{2.35}$$

$$\sqrt{2}z_3 = R_3 \, e^{i\theta_3}; \quad \sqrt{2}z_3{}^* = R_3 \, e^{-i\theta_3} \tag{2.36}$$

reduces the 6D integration (2.32) merely to $\pi^3 dR_1{}^2 dR_2{}^2 dR_3{}^2$, since the θ_i -variables (*not* Euler angles!) are not involved in the integrands encountered, and just sum up to $(2\pi)^3$. The positive variables R_i, (i = 1,2,3), are related to the ξ_i, η_i variables by

$$\rho \equiv R_1^2 + R_2^2 + R_3^2 = \xi^2 + \eta^2 = 2z_i z_i{}^* \tag{2.37}$$

To convert the variables ω_i that appear in the integrals (2.28) in terms of the $R_{1,2,3}$ variables is a straightforward but tedious process which can be somewhat simplified in terms of the intermediate variables $\xi^2 - \eta^2$ and $2\xi.\eta$ which form a [2,1] representation [36] of S_3-symmetry at the 'quadratic' level. Now because of the full S_3 -symmetry of the 6D integral (2.32), together with the (fortunate) circumstance of equal mass quarks in the problem on hand, the integrand as a whole is S_3 -symmetric which permits the following simplification: Each of the quantities $\hat{p}_i^2$ and $\hat{q}_i^2$ inside (2.32) can be expanded as

$$\hat{p}_{1,2}^2 = \rho/2 + (\xi^2 - \eta^2)/4 \pm \sqrt{3}\xi/\eta/2; \quad \hat{p}_3^2 = \rho/2 - (\xi^2 - \eta^2)/2 \tag{2.38}$$

$$\hat{q}_{12}^2 = 3\xi^2/4 = \rho/2 + (\xi^2 - \eta^2)/4 \tag{2.39}$$

In all these terms the principal quantity is $\rho/2$, while the resultant effects of the mixed-symmetric corrections will show up only in the *fourth* order, etc. In the present case of equal mass kinematics it is a good approximation to neglect the latter terms, as has also been found for the qqq mass spectral results [15], so that all quantities are expressed in terms of ρ only :

$$\omega_{1,2,3} \approx \omega_{12} \approx \omega_\rho; \quad \omega_\rho{}^2 \equiv m_q^2 + \rho/2 \tag{2.40}$$

$$D_{12} \approx 2(M - \omega_\rho)[\omega_\rho^2 - (M - \omega_\rho)^2/4]. \tag{2.41}$$

The rest of the integration is expressed entirely in terms of the ρ-variable, with the resultant 6D measure given by

$$\int d^3\hat{p}_3 d^3\hat{q}_{12} F(\rho) = (\pi\sqrt{3}/2)^3 \int \rho^2 d\rho/2 F(\rho) \tag{2.42}$$

These considerations suffice for evaluating the integrals I_1 and I_2 whose resultant value is now given for $e_i = 1/3$ by :

$$D_{12}^2(I_1+I_2) = [m_q^2 + m_q(M-\omega_\rho) + (M-\omega_\rho)^2/4] \times (M-\omega_\rho)^3/\omega_\rho + D_{12}(2m_q+M-\omega_\rho) \quad (2.43)$$

Substitution in (2.25) yields N_B directly. The numerical values are given collectvely at the end of Sec.4.

3   Strong SU(2) Mass Difference for the Nucleon

This calculation is on almost identical lines to Sec.2, except for the substitution $ie_1\gamma_\mu$ to $-\delta m \tau_3{}^{(1)}/2$ in figs.1(a,b) to give figs.2(a,b) which represent the effect of insertion of a 2-point vertex in a quark line. Indeed we can directly start from the counterpart of eq.(2.15) which gives the 'strong' mass shift as:

$$
\begin{aligned}
i\delta M_{st} &= \sum_{123} \int d^4q_{12} d^4p_3 \frac{\Gamma_3{}^*\Gamma_3}{2\Delta_2\Delta_3} \times [<\phi'|(23)'(1)'|\phi'> + \frac{1}{3} <\phi"|(23)"_{\nu\lambda}(1)"_{\nu\lambda}|\phi" >] \\
&\quad + (1 \Leftrightarrow 2) \quad\quad\quad (3.1)
\end{aligned}
$$

where we have now employed eq.(2.21) for the isospin factors, and the counterparts of (2.16) and (2.18) are respectively

$$(1)' = \bar{U}(P)S_F(p_1)[-\delta m\tau_3{}^{(1)}/2]S_F(p_1)U(P); \quad\quad (3.2)$$

$$(1)"_{\nu\lambda} = \bar{U}(P)i\hat{\gamma}_\nu\gamma_5 S_F(p_1)[-\delta m\tau_3{}^{(1)}/2]S_F(p_1)i\gamma_5\hat{\gamma}_\lambda U(P) \quad\quad (3.3)$$

while the definitions (2.19) and (2.20) remain unaltered. As a result, eq.(2.24) remains valid, while the counterpart of (2.23) becomes

$$(1)'\theta_{\nu\lambda} = -3(1)"_{\nu\lambda} = [2m_q(m_q + M\nu_1) - \Delta_1]\theta_{\nu\lambda}(-\delta m/2)/\Delta_1^2 \quad\quad (3.4)$$

Carrying out the $d\sigma_{12}$-integration, the result for δM_{st} is now given by the counterpart of (2.26), viz.,

$$\delta M_{st} = 3N_B^2 \int d^3\hat{p}_3 \frac{(m_q + \omega_3)}{2\omega_3} \times \int d^3\hat{q}_{12} D_{12}{}^2\phi^2[J_1 + J_2](-\delta m\tau_3/6) \quad\quad (3.5)$$

in the form of an isospin operator "τ_3" for the *nucleon*, where we have represented the effect of $\sum_{123}$ by a factor of "3", and

$$
\begin{aligned}
D_{12}^2[J_1 + J_2] &= \frac{m_q}{\omega_\rho}[(m_q + \frac{1}{2}(M-\omega_\rho))(M-\omega_\rho)[2\omega_\rho^2 + m_q(M-\omega_\rho)] \\
&\quad + (M-\omega_\rho)^2[m_q + (M-\omega_\rho)/2]^2 + \Delta_{12}(m_q^2 - \omega_\rho^2) \\
&\quad + (m - \omega_\rho)(m_q + (M-\omega_\rho)/2)(\omega_\rho^2 + m_q(M-\omega_\rho)/2) \\
&\quad + \Delta_{12}^2(1 - m_q(M-\omega_\rho)/\omega_\rho^2)/2] \quad\quad (3.6)
\end{aligned}
$$

as the exact counterpart of (2.43) under the same approximation. It is seen from (3.5) that the *difference* $(n - p)$ is *positive*.

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4 E.M. Mass Difference for the Nucleon

The diagrams for the e.m. mass difference are given by figs.3 (I,II,III) for a proton (uud) configuration to illustrate the underlying topology in accordance with the roles of the 'active' and 'spectator' quarks in turn, as explained in Sec.2. In each of these diagrams, two internal quark lines are joined by a photon line. The e.m. vertex at quark $\#i$ has the strength $e[1 + 3\tau_3^{(i)}]/6$ from which the isospin matrix elements of a product of *two* such factors (shown for fig.3.III) have the forms

$$< \phi'; \phi" |(1 + 3\tau_3^{(1)})/6 \times (1 + 3\tau_3^{(1)})/6|\phi'; \phi" > \tag{4.1}$$

for the proton (uud) cnfiguration shown in III with $\#3$ as spectator, but in a basis (1;23) (which is consistent with the spin basis, eqs.(2.3-4)), corresponding to fig.1a, viz.[37,39]:

$$|\phi' > = u_1(u_2d_3 - u_3d_2)/\sqrt{2}; \quad |\phi" > = (-2d_1u_2u_3 + u_1d_2u_3 + u_1u_2d_3)/\sqrt{6} \tag{4.2}$$

We note in parentheses that in fig.3.III, the interchange of the two 'active' quarks $\#1$ and $\#2$ does *not* give a new configuration, *unlike* in figs.1 and 2; ((a) versus (b) configurations).

It is now easy to check that the matrix elements $<>'$ and $<>$" of (4.1) are 1/9 and $-1/9$ for the *proton* configuration. After doing the corresponding neutron case, the two results may be combined in the single operator forms [39]:

$$< . >' = (1 + 3\tau_3)/36; \quad < . >" = (1 - 5\tau_3)/36 \tag{4.3}$$

where τ_3 is the isospin operator for the nucleon as a whole [see eq.(3.5)], to be sandwiched between the neutron and proton states. The resultant isospin factor is then

$$e^2[< . >' + < . >"]/2 = e^2(1 - \tau_3)/36 \Rightarrow -e^2\tau_3/36 \tag{4.4}$$

After this book-keeping on the charge factors we can drop the isospin d.o.f. $|\phi >$ from the qqq wave function and, on the basis of the equality of the $(.)'$ and $(.)"$ contributions (2.22-23) for the spin matrix elements, it is enough to work with the $(.)'$ type to represent the full effect. Collecting these details, the net isospin contribution to the e.m. $(n - p)$ mass difference is just $e^2/18$, which (of course) comes out with the *correct* (negative) sign in the resultant e.m.contribution to the total $n - p$ difference after all the phase factors in the orbital-cum-spin space have been taken into account. The complete e.m. self energy of the nucleon (with operator τ_3), with fig.3.III as the prototype, is now given by

$$\delta M^\gamma = \sum_{123}[-e^2\tau_3/36]/(2\pi)^4 \int d^4p_3 d^4q_{12} d^4q_{12}' \Gamma_3^* \Gamma_3' k^{-2} \times [23]_\mu'[1]_\mu'/(\Delta_3 \Delta_2 \Delta_2') \tag{4.5}$$

where the various momentum symbols are as shown in fig.3, with the primed quantities referring to the vertex on the right, but otherwise written in the same convention as in eqs.(2.5-10). The symbols within square brackets are analogous to (2.16-19):

$$[1]_\mu' = \bar{U}(P')S_F(p_1')i\gamma_\mu S_F(p_1)U(P); \quad (P' = P) \tag{4.6}$$

$$[23]_\mu' = \frac{Tr}{8M^2}[C^{-1}\gamma_5(M - i\gamma.P')(m_q - i\gamma.p_2')i\gamma_\mu(m_q - i\gamma.p_2)(M - i\gamma.P)\gamma_5(m_q + i\gamma.p_3)C] \tag{4.7}$$

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And the product of (4.6) and (4.7) works out as

$$
\begin{aligned}
ME \;\equiv\; & \frac{(m_q + \omega_3)}{\Delta_1 \Delta_1'}[-(\Delta_1 + \Delta_1' - k^2)(\Delta_2 + \Delta_2' - k^2)/4 \\
& -(\Delta_1 + \Delta_1' - k^2)(m_q\omega_2 + m_q\omega_2' + 2\omega_2\omega_2')/2 \\
& -(\Delta_2 + \Delta_2' - k^2)(m_q\omega_1 + m_q\omega_1' + 2\omega_1\omega_1')/2 \\
& -(\Delta_1 + \Delta_2)(m_q + \omega_1')(m_q + \omega_2')/2 - (\Delta_1' + \Delta_2')(m_q + \omega_1)(m_q + \omega_2)/2 \\
& -(\Delta_1' + \Delta_2)(m_q + \omega_1)(m_q + \omega_2')/2 - (\Delta_1 + \Delta_2')(m_q + \omega_1')(m_q + \omega_2)/2 \\
& +(m_q^2 + \frac{1}{2}(P - p_3 - k)^2)[(m_q + \omega_1)(m_q + \omega_2') + (m_q + \omega_1')(m_q + \omega_2) \\
& +(m_q + \omega_1)(m_q + \omega_2) + (m_q + \omega_1')(m_q + \omega_2')]]
\end{aligned}
\tag{4.8}
$$

Some features of this "master" expression may be noted. There is a 'natural factorization'in the variables q_{12} and q_{12}', except for the photon propagator k^{-2}, $(k = q_{12} - q_{12}')$. Further, the two blobs are connected together by the 'spectator'variable p_3 which is on the mass shell due to the presence of $\Gamma_3^* \Gamma_3'$ in eq.(4.3).

The time-like (pole) integrations over each of $d\sigma_{12}$ and σ_{12}' can be carried out *exactly* a la (2.28) and its derivatives, since the 3D vertex function $D_{12}\phi$ in Γ_3 does not involve σ_{12}, etc. After this step $\hat{q}_{12}$, $\hat{q}_{12}'$ and $\hat{p}_3$ are the 'right' 3D variables for the 'triple integration' whose essential logic may be stated as follows. The main strategy is to decouple the $\hat{q}$ and $\hat{q}'$ variables from the photon propagator k through the following device [28]:

Since k is basically space- like, it is a good approximation to replace k^{-2} by $\hat{k}^{-2}$ which equals $(\hat{q}_{12} - \hat{q}_{12}')^2$, and drop the angular correlation in the two $\hat{q}$- momenta (since the error in this neglect is zero in the first order [28]). Next we use the inequality [28]

$$
(a^2 + b^2)^{-1} \leq (2ab)^{-1}; \quad a \to |\hat{q}_{12}|, etc
\tag{4.9}
$$

which ensures the necessary factorizability in the q-variables.In principle the corrections to this inequality can be calculated since the neglected term is approximately equal to $-(a - b)^2/(4a^2b^2)$ which is still factorizable, but this refinement is unnecessary in view of the smallness of the e.m. effect itself. After this simplification the rest of the integration procedure is straightforward since the $\hat{q}$ and $\hat{q}'$ integrations can be done analytically, and only a 1D integration over $|p_3|$ remains for numerical evaluation. The necessary expressions are collected in Appendix A and the numerical results for all contributions are given as under.

The key parameters are the quark mass m_q and the size parameter β^2, the latter being determined dynamically through the chain of eqs.(2.11-14). As noted in Sec.1 already, the mass m_q which is usually called the 'constituent' mass, should be viewed as the sum of the (flavour independent) 'mass function' m(p) for small p, plus a small "current mass" m_c, in the spirit of Politzer additivity [22]. The mass function m(p) was generated in this BSE-cum-SDE framework through a Dynamical Chiral Symmetry Breaking mechanism in a non perturbative fashion [23]. Also from some related quark-loop calculations with $q\bar{q}$ mesons in recent times [24,28], it was found that for such 'low energy' processes the mass function m(p) is rather well approximated by m(0), so that [22], $m_q = m(0) + m_c$. Therefore the $d-u$ mass difference is the *same* at the 'constituent' or at the 'current' levels, and this is what has been denoted by δm in the text (figs.2). Its smallness compared to m_q justifies its neglect in all the functions except where it appears explicitly, viz., fig.2. We

take its value at $\delta m = 4 MeV$, as in related calculations [24,28], while the other quantities are predetermined from $q\bar{q}$ [14] and qqq [15] spectroscopy:

$$m_q = 265 MeV; \quad \beta^2(N) = 0.052 GeV^2 \tag{4.10}$$

so that there are *no free* parameters in the entire calculation. The results from Secs.2-4 are now summarized for (n-p) as :

$$N_B^{-2} = 5.5209 \times 10^{-4} GeV^{-10}; \quad [e_i = 1/3] \tag{4.11}$$

$$\delta M_{st} = +1.7134 MeV; \quad \delta M^\gamma = -0.4396 MeV. \tag{4.12}$$

Hence

$$\delta M(net) = +1.28 MeV; \quad (vs.1.29 MeV : Expt) \tag{4.13}$$

which is the principal result of this investigation, but subject to possible gauge invariance corrections [32]; see Appendix C and discussion in sec.5.3.

5 Discussion, Summary and Conclusion

This calculation fills up an important gap in the two-tier BSE formalism under 3D kernel support (CIA) for a simultaneous investigation of spectra and transition amplitudes of both $q\bar{q}$ and qqq varieties under a single umbrella [8,12].

5.1 Recapitulation

To recapitulate the main points, the (first stage) 3D reductions of both the 2-body and 3-body BSE's had yielded good agreement with the respective spectra [14,15], with a common set of parameters $C_0 = 0.27$ and $\omega_0 = 158 MeV$ characterizing the non-perturbative gluon propagator, since the constituent mass m_q for spectroscopy [14,15], is essentially the dynamical mass function $m(p)$ in the low momentum limit [22, 23].

More substantial tests of the formalism have come from the (second stage) reconstruction of the 4D hadron-quark vertex function which carries the non-perturbative off-shell information in a *closed* form. This exercise was initially confined to the meson-$q\bar{q}$ vertex function whose exact reconstruction [8] had led to several useful results from 4D loop integrals for hadronic and e.m. transition amplitudes [8,16], to like integrals probing the momentum dependence of the quark mass function $m(p)$ which is the 'chiral' limit ($M_\pi = 0$) [17,21,23] of the pion-quark vertex function. Indeed $m(p)$ acts as the form factor for loop integrals determining the vacuum to vacuum transitions, and is found to predict correctly several condensates, from the basic $< q\bar{q} >$ [23] to 'induced' condensates [40], *under one roof*. Further tests of the hadron-quark vertex function have come from SU(2) breaking effects like $\rho - \omega$ mixing [24] and mass splittings in pseudoscalar mesons [28], with only one additional parameter representing the $d - u$ mass difference.

The last link in our two-tier formalism has been a reconstruction of the 4D baryon-qqq vertex function on $q\bar{q}$ lines [8], to evaluate 3-body loop integrals. Although conjectured some time ago [12], a rigorous derivation [29] via Green's functions is outlined in Appendix B, in preparation for our main task: the $n - p$ mass difference on identical lines to the $q\bar{q}$ case [28], viz., strong SU(2) breaking (fig 2) and e.m.contribution (fig 3), since the qqq vertex function is entirely determined by the same (gluon exchange) dynamics [7,23]

as $q\bar{q}$. Strong SU(2) breaking is due to $d - u$ mass difference, a la Weinberg [41], and more references on the physics of the problem may be found in [28]. The point to stress is that no free parameters are involved, so that the final value (4.13), although a single number, must *not* be treated as an isolated quantity, but as an integral part of a much bigger package.

The 'QCD' status of this 3D-4D BSE formalism [8] viv-a-vis chiral perturbation theory [27] has already been explained in Sec.1: the gluon exchange character of the pairwise $q\bar{q}$ or qq interactions lends them a natural chiral invariance property at the input Lagrangian level with 'current' quarks. Thus the 'constituent' mass is *not* an input, but emerges as the low momentum limit of the dynamical mass function $m(p)$ that characterizes the quark propagators appearing in the 2- and 3- body BSE's, as a result of $DB\chi S$ [21, 17, 19, 23], since the 'current' masses of u, d quarks give only a small additive contribution [22]. The empirical aspect of the gluon propagator concerns only its *non-perturbative* regime which often requires separate parametrization even in more orthodox BSE-SDE formulations [18]. In the present formulation, its explicit parametrization with two constants C_0 and ω_0) [23] symbolizes a 'closed form' representation of non-perturbative effects in the derived hadron-quark vertex function, but the returns are rich, especially the interlinkage of 4D loop integrals for different transition amplitudes [12,23,24,28], with the 3D BSE structures relevant to spectroscopy [14,15].

In contrast, Chiral Perturbation Theory [27] has an explicit QCD content, but relies more heavily on a perturbative treatment, as revealed by expansions in powers of small momenta and "current" masses m_c [27] for a systemic derivation of the low energy structure of the Green's function in QCD [27]. It is a powerful method, highly successful in predicting items like ground state masses *as well as* their splittings, but its lack of a closed form representation prevents an equally successful prediction of 'soft' QCD effects in enough details, such as the momentum dependence of the mass function, or of hadron-quark vertex functions in general, and other observable effects such as *L-excited* spectra.

5.2   Comparison with QCD-Sum Rules

For a comparison with QCD-Sum Rules, it is useful to start by mentioning our neglect of the condensate contribution inserted in the internal quark lines, vide Fig 1c of [28], on the ground that it was found to be small in the 2-body case within the same BSE framework [28]. This contrasts sharply with the corresponding QCD-SR scenario [26] wherein the condensate contributions are the primary source of non-perturbative effects, as confirmed by explicit calculations [42]. Even more surprising is the *inversion* of the effect of the $d - u$ mass difference on the $n - p$ mass difference vis-a-vis the traditional low energy wisdom which requires $d - u > 0$ to make $n - p > 0$, as was the original motivation behind the famous Weinberg proposal [41]. (Incidentally our BSE result conforms to the Weinberg picture [41] and hence opposite of QCD-SR [42]). Indeed QCD-SR must rely heavily on the condensate contributions to compensate for the (negative) effect of the $d - u$ mass difference, and leave a balance [42]. The e.m. effect too makes a comparable contribution to QCD-SR [42]. On the other hand the BSE approach, eq.(4.12), seems to predict only a modest e.m. effect which is about a fourth of, and of opposite sign to, the strong SU(2) breaking effect which dominates the entire scenario.

What could account for such a sharp division between the two approaches? Perhaps

the reason should be sought in the very difference in the philosophy behind their respective premises: QCD-SR is basically a perturbative QCD approach designed from the high energy end, with the 'twist' terms (condensates) representing non-perturbative corrections of successively higher orders. In contrast the BSE is an intrinsically non-perturbative approach built from the low energy (spectroscopy) end, with the hard gluon exchange added perturbatively. The role of condensates in BSE is effectively subsumed in the constituent (non-perturbative) quark mass as well as the hadron-quark vertex function, so that any further condensate effects (fig 1c of [28]) in such a non-perturbative scenario can at best be residual [28]. This is not the case in QCD-SR wherein the quark condensates with their isospin splittings are the dominant source of non-perturbative effects on the $n - p$ splitting [42]. The two scenarios are thus largely complementary, QCD-SR being rooted in hard QCD, and BSE-SDE in soft QCD premises respectively.

5.3   Problem of Gauge Invariance of E.M. Effects

Finally we come to a more vulnerable aspect of this investigation in company with the earlier study [28], viz., the lack of gauge invariance of the e.m. contribution (fig.(3)). Mercifully the e.m. contributions in both the meson [28] and baryon (present) cases are about a fourth of the $u - d$ effect so as hopefully not to upset the overall stability of our result (4.13), but the need for a proper assessment of the g.i. corrections can hardly be overestimated. A general method for g.i. two-point functions for $Q\bar{q}$ systems has been given in [32], but it is not directly adaptable to extended vertex functions with arbitrary form factors, such as in the present situation. To handle these structures requires a different kind of strategy, a full-fledged formulation of which, is beyond the scope of this (already long) paper, and is best left to a separate communication. Nevertheless, as noted in Sec.1, we have made a beginning in this paper by outlining the main steps of the derivation of g.i. corrections for a typical two-body (kaon) case, in a simple and straightforward fashion, which amounts to the replacement of various momenta p_i involved in the hadron-quark vertex functions by $(p_i - e_i A)$, and expanding in powers of the e.m. field, with a view to calculate the additional diagrams a la [32]. Hopefully this method of generating e.m. gauge corrections is general enough to apply to other situations in which arbitrary momentum dependent hadron-quark vertex functions are involved. For the present situation of QED gauge corrections, the main steps are sketched in Appendix C, and the resultant correction to fig 1b of [28] (i.e. fig 1a of [32]) is estimated to be $-0.612 MeV$, which is $3/5$ times, and of the same sign as, the e.m. value $-1.032 MeV$ for the mass difference $K^- - \bar{K}^0$ [28]. If this result is taken as a rough indication of the g.i. effect expected for the nucleon case on hand, it would mean a downward revision of the value (4.13) to about $1 MeV$. However this is only a provisional estimate, pending a regular qqq calculation in the future.

5.4   Conclusion

To conclude, the principal motivation for this investigation, has been to demonstrate the practical feasibility of such realistic quark-loop calculations for the relativistic 3-quark problem within a full-fledged (BS) dynamical framework whose basic parameters are linked all the way to spectroscopy. The present calculation indeed suggests that not only quark-loops involving mesons [8,23,24,28] but even those involving the (less trivial)

qqq baryon are amenable to a similar degree of dynamical sophistication without excessive efforts, so that it makes sense to speak of an effective "4-fermion coupling" for both $q\bar{q}$ and qq pairs within a common parametric framework. This is somewhat remiscient of Bethe's "second principle" theory, originally suggested at the two-nucleon level of nuclear forces, now reinterpreted at the quark level, with a simple extension to include the antiquark in the dynamical description. (This extension would not make sense at the NN level, since the NN and $N\bar{N}$ forces are very different from each other). Indeed such a dynamics had been strongly suggested (with concrete examples) in a perspective review not too long ago [43], but it seemed to have gone largely by default, as evidenced by a strong tendency in the contemporary literature to continue to rely on "ad-hoc form factors" [44] to simulate the vertex functions, instead of generating them dynamically. Hopefully, some efforts in this direction have been recently in evidence [45], using the Nambu Jonalasino model [21] of contact 4-fermion interactions (although such contact interaction models are probably too simple to realistically simulate confinement [29]). It is to be hoped that Bethe's "second principle" perspective will be upheld by such investigations, until such time as a fully satisfactory solution to QCD is forthcoming.

Appendix A: Evaluation of the Integral (4.5)

The master expression (4.8) after being substituted in the full e.m. self energy contribution (4.5) is integrated over each $d\sigma_{12}$ and $d\sigma_{12}'$. The final result is

$$\delta M^{\gamma} = \sum_{123} \frac{2e^2}{9}\tau_3 \int \frac{(m_q + \omega_3)}{2\omega_3} \frac{d^3\hat{p}_3}{4\pi} \frac{d^3\hat{q}_{12}}{4\pi} \frac{d^3\hat{q}_{12}'}{4\pi} \frac{1}{2\hat{q}_{12}2\hat{q}_{12}'} \times$$

$$F(\hat{q}_{12}, \hat{q}_{12}', \hat{p}_3) exp(-\frac{2}{3}[\hat{q}_{12}^2 + \hat{q}_{12}'^2 + 3\hat{p}_3^2]/\beta^2) \qquad (A.1)$$

where

$$F(\hat{q}_{12}, \hat{q}_{12}', \hat{p}_3) =$$

$$(m_q + \omega_{12})(m_q + \omega_{12}')k^2 + (M\omega_3 - \frac{1}{2}(M^2 - m_q^2))[(m_q + \omega_{12})^2$$

$$+ (m_q + \omega_{12}')^2 + (m_q + \omega_{12})(m_q + \omega_{12}') - (m_q + \omega_{12})^2 D_{12}'/2\omega_{12}'$$

$$- (m_q + \omega_{12}')^2 D_{12}/2\omega_{12} - (m_q + \omega_{12})(m_q + \omega_{12}')[D_{12}/2\omega_{12}$$

$$+ D_{12}'/2\omega_{12}']k^2[(m_q + 2\omega_{12})(m_q + 2\omega_{12}') - m_q^2]/2$$

$$- [(m_q + 2\omega_{12})(m_q + 2\omega_{12}') - m_q^2][D_{12}/2\omega_{12} + D_{12}'/2\omega_{12}']/2$$

$$- \frac{1}{8}\frac{D_{12}D_{12}'}{\omega_{12}\omega_{12}'} + k^4 - k^2[D_{12}/\omega_{12} + D_{12}'/\omega_{12}'] \qquad (A.2)$$

Using eq.(4.9), the integration over $\hat{q}_{12}$ and $\hat{q}_{12}'$ can be done independently of each other, and thus can be written in a compact notation as follows

$$\delta M^{\gamma} = \sum_{123}(+\frac{2}{9}e^2\tau_3) \int \hat{p}_3^2 d\hat{p}_3 \frac{(m_q + \omega_3)}{2\omega_3} F_1 \; e^{[-\hat{p}_3^2/\beta^2]} \qquad (A.3)$$

where

$$
\begin{aligned}
F_1 \;=\;& J_{11}J_{11} + [M\omega_3 - \frac{1}{2}M^2 + \frac{1}{2}m_q^2](2J_{20}J_{00} + 2J_{10}J_{10}) - 2J_{20}I_{00} \\
& -2J_{10}I_{10} + J_{11}{}'J_{11}{}'/2 - J_{01}J_{01}m_q^2/2 - I_{10}{}'J_{10}{}' \\
& +m_q^2 I_{00}J_{00} - I_{00}I_{00}/2 - J_{02}J_{02}/4 + I_{01}I_{01}
\end{aligned}
\tag{A.4}
$$

and with $(n = 0, 1, 2; m = 0, 1, 2)$,

$$
J_{nm}; I_{nm} = 2^{-1/2} \int \hat{q}_{12} d\hat{q}_{12} \; e^{(-\frac{2}{3}\hat{q}_{12}^2/\beta^2)} [\sqrt{2}\hat{q}_{12}]^m (m_q + \omega_{12})^n [1; \frac{1}{2}D_{12}/\omega_{12}];
\tag{A.5}
$$

$$
J_{nm}{}'; I_{nm}{}' = 2^{-1/2} \int \hat{q}_{12} d\hat{q}_{12} \; e^{(-\frac{2}{3}\hat{q}_{12}^2/\beta^2)} [\sqrt{2}\hat{q}_{12}]^m (m_q + 2\omega_{12})^n [1; \frac{1}{2}D_{12}/\omega_{12}];
\tag{A.6}
$$

Appendix B: Derivation of qqq Vertex Fn, Eq.(2.5)

B.1: Method of Green's Functions

We outline here some essential steps leading to a formal derivation of eq.(2.5) which was written down in a semi-intuitive fashion in [12]. To that end we shall employ the method of Green's functions for 2- and 3- particle scattering *near the bound state pole*, since the inhomogeneous terms are not relevant for our purposes. For simplicity we shall consider identical *spinless* bosons, with pairwise BS kernels under CIA conditions [8], first for the 2-body case for calibration, and then for the 3-body system.

B.2: Two-Quark Green's Function

Apart from some results already goven in the text, we shall use the notation and phase conventions of [8,12] for the various quantities (momenta, propagators, etc). The 4D qq Green's function $G(p_1p_2; p_1'p_2')$ near a *bound* state satisfies a 4D BSE without the inhomogeneous term, viz. [8,12],

$$
i(2\pi)^4 G(p_1p_2; p_1'p_2') = \Delta_1^{-1}\Delta_2^{-1} \int dp_1{}'' dp_2{}'' K(p_1p_2; p_1{}''p_2{}'') G(p_1{}''p_2{}''; p_1'p_2')
\tag{B.2.1}
$$

where

$$
\Delta_1 = p_1{}^2 + m_q{}^2,
\tag{B.2.2}
$$

and m_q is the mass of each quark. Now using the relative 4- momentum $q = (p_1 - p_2)/2$ and total 4-momentum $P = p_1 + p_2$ (similarly for the other sets), and removing a δ-function for overall 4-momentum conservation, from each of the G- and K- functions, eq.(B.2.1) reduces to the simpler form

$$
i(2\pi)^4 G(q.q') = \Delta_1^{-1}\Delta_2^{-1} \int d\hat{q}'' M d\sigma'' K(\hat{q}, \hat{q}'') G(q'', q')
\tag{B.2.3}
$$

where $\hat{q}_\mu = q_\mu - \sigma P_\mu$, with $\sigma = (q.P)/P^2$, is effectively 3D in content (being orthogonal to P_μ). Here we have incorporated the ansatz of a 3D support for the kernel K (independent of σ and σ'), and broken up the 4D measure dq'' arising from (2.1) into the

product $d\hat{q}''Md\sigma''$ of a 3D and a 1D measure respectively. We have also suppressed the 4-momentum P_μ label, with $(P^2 = -M^2)$, in the notation for $G(q.q')$.

Now define the fully 3D Green's function $\hat{G}(\hat{q}, \hat{q}')$ as [29]

$$\hat{G}(\hat{q}, \hat{q}') = \int \int M^2 d\sigma d\sigma' G(q, q') \tag{B.2.4}$$

and two (hybrid) 3D-4D Green's functions $\tilde{G}(\hat{q}, q')$, $\tilde{G}(q, \hat{q}')$ as

$$\tilde{G}(\hat{q}, q') = \int Md\sigma G(q, q'); \tilde{G}(q, \hat{q}') = \int Md\sigma' G(q, q'); \tag{B.2.5}$$

Next, use (B.2.5) in (B.2.3) to give

$$i(2\pi)^4 \tilde{G}(q, \hat{q}') = \Delta_1^{-1}\Delta_2^{-1} \int dq'' K(\hat{q}, \hat{q}'') \tilde{G}(q'', \hat{q}') \tag{B.2.6}$$

Now integrate both sides of (B.2.3) w.r.t. $Md\sigma$ and use the result [8]

$$\int Md\sigma \Delta_1^{-1}\Delta_2^{-1} = 2\pi i D^{-1}(\hat{q}); \quad D(\hat{q}) = 4\hat{\omega}(\hat{\omega}^2 - M^2/4); \quad \hat{\omega}^2 = m_q^2 + \hat{q}^2 \tag{B.2.7}$$

to give a 3D BSE w.r.t. the variable $\hat{q}$, while keeping the other variable q' in a 4D form:

$$(2\pi)^3 \tilde{G}(\hat{q}, q') = D^{-1} \int d\hat{q}'' K(\hat{q}, \hat{q}'') \tilde{G}(\hat{q}'', q') \tag{B.2.8}$$

Now a comparison of (B.2.3) with (B.2.8) gives the desired connection between the full 4D G-function and the hybrid $\tilde{G}(\hat{q}, q')$-function:

$$2\pi i G(q, q') = D(\hat{q})\Delta_1^{-1}\Delta_2^{-1} \tilde{G}(\hat{q}, q') \tag{B.2.9}$$

Again, the symmetry of the left hand side of (B.2.9) w.r.t. q and q' allows us to write the right hand side with the roles of q and q' interchanged. This gives the dual form

$$2\pi i G(q, q') = D(\hat{q}')\Delta_1'^{-1}\Delta_2'^{-1} \tilde{G}(q, \hat{q}') \tag{B.2.10}$$

which on integrating both sides w.r.t. $Md\sigma$ gives

$$2\pi i \tilde{G}(\hat{q}, q') = D(\hat{q}')\Delta_1'^{-1}\Delta_2'^{-1} \hat{G}(\hat{q}, \hat{q}'). \tag{B.2.11}$$

Substitution of (B.2.11) in (B.2.9) then gives the symmetrical form

$$(2\pi i)^2 G(q, q') = D(\hat{q})\Delta_1^{-1}\Delta_2^{-1} \hat{G}(\hat{q}, \hat{q}') D(\hat{q}')\Delta_1'^{-1}\Delta_2'^{-1} \tag{B.2.12}$$

Finally, integrating both sides of (B.2.8) w.r.t. $Md\sigma'$, we obtain a fully reduced 3D BSE for the 3D Green's function:

$$(2\pi)^3 \hat{G}(\hat{q}, \hat{q}') = D^{-1}(\hat{q} \int d\hat{q}'' K(\hat{q}, \hat{q}'') \hat{G}(\hat{q}'', \hat{q}') \tag{B.2.13}$$

Eq.(B.2.12) which is valid near the bound state pole (since the inhomogeneous term has been dropped for simplicity) expresses the desired connection between the 3D and 4D forms of the Green's functions; and eq(B.2.13) is the determining equation for the 3D

form. A spectral analysis can now be made for either of the 3D or 4D Green's functions in the standard manner, viz.,

$$G(q, q') = \sum_n \Phi_n(q; P)\Phi_n^*(q'; P)/(P^2 + M^2) \tag{B.2.14}$$

where Φ is the 4D BS wave function. A similar expansion holds for the 3D G-function $\hat{G}$ in terms of $\phi_n(\hat{q})$. Substituting these expansions in (B.2.12), one immediately sees the connection between the 3D and 4D wave functions in the form:

$$2\pi i\Phi(q, P) = \Delta_1^{-1}\Delta_2^{-1}D(\hat{q})\phi(\hat{q}) \tag{B.2.15}$$

whence the BS vertex function becomes $\Gamma = D \times \phi/(2\pi i)$ as found in [8]. We shall make free use of these results, taken as qq subsystems, for our study of the qqq G-functions in Sections 3 and 4.

B.3: 3D Reduction of the BSE for 3-Quark G-function

As in the two-body case, and in an obvious notation for various 4-momenta (without the Greek suffixes), we consider the most general Green's function $G(p_1p_2p_3; p_1'p_2'p_3')$ for 3-quark scattering *near the bound state pole* (for simplicity) which allows us to drop the various inhomogeneous terms from the beginning. Again we take out an overall delta function $\delta(p_1 + p_2 + p_3 - P)$ from the G-function and work with *two* internal 4-momenta for each of the initial and final states defined as follows [12]:

$$\sqrt{3}\xi_3 = p_1 - p_2 ; \quad 3\eta_3 = -2p_3 + p_1 + p_2 \tag{B.3.1}$$

$$P = p_1 + p_2 + p_3 = p_1' + p_2' + p_3' \tag{B.3.2}$$

and two other sets ξ_1, η_1 and ξ_2, η_2 defined by cyclic permutations from (B.3.1). Further, as we shall consider pairwise kernels with 3D support, we define the effectively 3D momenta $\hat{p}_i$, as well as the three (cyclic) sets of internal momenta $\hat{\xi}_i, \hat{\eta}_i$, (i = 1,2,3) by [12]:

$$\hat{p}_i = p_i - \nu_i P ; \quad \hat{\xi}_i = \xi_i - s_i P ; \quad \hat{\eta}_i - t_i P \tag{B.3.3}$$

$$\nu_i = (P.p_i)/P^2 ; \quad s_i = (P.\xi_i)/P^2 ; \quad t_i = (P.\eta_i)/P^2 \tag{B.3.4}$$

$$\sqrt{3}s_3 = \nu_1 - \nu_2 ; \quad 3t_3 = -2\nu_3 + \nu_1 + \nu_2 \quad (+\text{cyclic permutations}) \tag{B.3.5}$$

The space-like momenta $\hat{p}_i$ and the time-like ones ν_i satisfy [12]

$$\hat{p}_1 + \hat{p}_2 + \hat{p}_3 = 0 ; \quad \nu_1 + \nu_2 + \nu_3 = 1 \tag{B.3.6}$$

Strictly speaking, in the spirit of covariant instantaneity, we should have taken the relative 3D momenta $\hat{\xi}, \hat{\eta}$ to be in the instantaneous frames of the concerned pairs, i.e., w.r.t. the rest frames of $P_{ij} = p_i + p_j$; however the difference between the rest frames of P and P_{ij} is small and calculable [12], while the use of a common 3-body rest frame ($P = 0$) lends considerable simplicity and elegance to the formalism.

We may now use the foregoing considerations to write down the BSE for the 6-point Green's function in terms of relative momenta, on closely parallel lines to the 2-body case. To that end note that the 2-body relative momenta are $q_{ij} = (p_i - p_j)/2 = \sqrt{3}\xi_k/2$, where (ijk) are cyclic permutations of (123). Then for the reduced qqq Green's function, when

the *last* interaction was in the (ij) pair, we may use the notation $G(\xi_k\eta_k; \xi_k'\eta_k')$, together with 'hat' notations on these 4-momenta when the corresponding time-like components are integrated out. Further, since the pair ξ_k, η_k is *permutation invariant* as a whole, we may choose to drop the index notation from the complete G-function to emphasize this symmetry as and when needed. The G-function for the qqq system satisfies, in the neighbourhood of the bound state pole, the following (homogeneous) 4D BSE for pairwise qq kernels with 3D support:

$$i(2\pi)^4 G(\xi\eta; \xi'\eta') = \sum_{123} \Delta_1^{-1}\Delta_2^{-1} \int d\hat{q}_{12}'' M d\sigma_{12}'' K(\hat{q}_{12}, \hat{q}_{12}'') G(\xi_3''\eta_3''; \xi_3'\eta_3') \quad \text{(B.3.7)}$$

where we have employed a mixed notation (q_{12} versus ξ_3) to stress the two-body nature of the interaction with one spectator at a time, in a normalization directly comparable with eq.(B.2.3) for the corresponding two-body problem. Note also the connections

$$\sigma_{12} = \sqrt{3}s_3/2; \quad \hat{q}_{12} = \sqrt{3}\hat{\xi}_3/2; \quad \hat{\eta}_3 = -\hat{p}_3, \quad etc \quad \text{(B.3.8)}$$

The next task is to reduce the 4D BSE (B.3.7) to a fully 3D form through a sequence of integrations w.r.t. the time-like momenta s_i, t_i applied to the different terms on the right hand side, *provided both* variables are simultaneously permuted. We now define the following fully 3D as well as mixed (hybrid) 3D-4D G-functions according as one or more of the time-like ξ, η variables are integrated out:

$$\hat{G}(\hat{\xi}\hat{\eta}; \hat{\xi}'\hat{\eta}') = \int\int\int\int dsdtds'dt' G(\xi\eta; \xi'\eta') \quad \text{(B.3.9)}$$

which is S_3-symmetric.

$$\tilde{G}_{3\eta}(\xi\hat{\eta}; \xi'\hat{\eta}') = \int\int dt_3 dt_3' G(\xi\eta; \xi'\eta'); \quad \text{(B.3.10)}$$

$$\tilde{G}_{3\xi}(\hat{\xi}\eta; \hat{\xi}'\eta') = \int\int ds_3 ds_3' G(\xi\eta; \xi'\eta'); \quad \text{(B.3.11)}$$

The last two equations are however *not* symmetric w.r.t. the permutation group S_3, since both the variables ξ, η are not simultaneously transformed; this fact has been indicated in eqs.(B.3.10-11) by the suffix "3" on the corresponding (hybrid) $\tilde{G}$-functions, to emphasize that the 'asymmetry' is w.r.t. the index "3". We shall term such quantities "S_3-indexed", to distinguish them from S_3-symmetric quantities as in eq.(B.3.9). The full 3D BSE for the $\hat{G}$-function is obtained by integrating out both sides of (B.3.7) w.r.t. the st-pair variables $ds_i ds_j' dt_i dt_j'$ (giving rise to an S_3-symmetric quantity), and using (B.3.9) together with (B.3.8) as follows:

$$(2\pi)^3 \hat{G}(\hat{\xi}\hat{\eta}; \hat{\xi}'\hat{\eta}') = \sum_{123} D^{-1}(\hat{q}_{12}) \int d\hat{q}_{12}'' K(\hat{q}_{12}, \hat{q}_{12}'') \hat{G}(\hat{\xi}''\hat{\eta}''; \hat{\xi}'\hat{\eta}') \quad \text{(B.3.12)}$$

This integral equation for $\hat{G}$ which is the 3-body counterpart of (B.2.13) for a qq system in the neighbourhood of the bound state pole, is the desired 3D BSE for the qqq system in a *fully connected* form, i.e., free from delta functions. Now using a spectral decomposition for $\hat{G}$

$$\hat{G}(\hat{\xi}\hat{\eta}; \hat{\xi}'\hat{\eta}') = \sum_n \phi_n(\hat{\xi}\hat{\eta}; P)\phi_n^*(\hat{\xi}'\hat{\eta}'; P)/(P^2 + M^2) \quad \text{(B.3.13)}$$

on both sides of (B.3.12) and equating the residues near a given pole $P^2 = -M^2$, gives the desired equation for the 3D wave function ϕ for the bound state in the connected form:

$$(2\pi)^3 \phi(\hat{\xi}\hat{\eta}; P) = \sum_{123} D^{-1}(\hat{q}_{12}) \int d\hat{q}_{12}'' K(\hat{q}_{12}, \hat{q}_{12}'') \phi(\hat{\xi}''\hat{\eta}''; P) \qquad (B.3.14)$$

Now the S_3-symmetry of ϕ in the $(\hat{\xi}_i, \hat{\eta}_i)$ pair is a very useful result for both the solution of (B.3.14) *and* for the reconstruction of the 4D BS wave function in terms of the 3D wave function (B.3.14), as is done in the subsection below.

B.4: Reconstruction of the 4D BS Wave Function

We now attempt to *re-express* the 4D G-function given by (B.3.7) in terms of the 3D $\hat{G}$-function given by (B.3.12), as the qqq counterpart of the qq results (B.2.12-13). To that end we adapt the result (B.2.12) to the hybrid Green's function of the (12) subsystem given by $\tilde{G}_{3\eta}$, eq.(B.3.10), in which the 3-momenta $\hat{\eta}_3, \hat{\eta}_3'$ play a parametric role reflecting the spectator status of quark #3, while the *active* roles are played by $q_{12}, q_{12}' = \sqrt{3}(\xi_3, \xi_3')/2$, for which the analysis of subsec.B.2 applies directly. This gives

$$(2\pi i)^2 \tilde{G}_{3\eta}(\xi_3 \hat{\eta}_3; \xi_3' \hat{\eta}_3') = D(\hat{q}_{12}) \Delta_1{}^{-1} \Delta_2{}^{-1} \hat{G}(\hat{\xi}_3 \hat{\eta}_3; \hat{\xi}_3' \hat{\eta}_3') D(\hat{q}_{12}') \Delta_1'{}^{-1} \Delta_2'{}^{-1} \qquad (B.4.1)$$

where on the right hand side, the 'hatted' G-function has full S_3-symmetry, although (for purposes of book-keeping) we have not shown this fact explicitly by deleting the suffix '3' from its arguments. A second relation of this kind may be obtained from (B.3.7) by noting that the 3 terms on its right hand side may be expressed in terms of the hybrid $\tilde{G}_{3\xi}$ functions vide their definitions (B.3.11), together with the 2-body interconnection between (ξ_3, ξ_3') and $(\hat{\xi}_3, \hat{\xi}_3')$ expressed once again via (B.4.1), but without the 'hats' on η_3 and η_3'. This gives

$$\begin{aligned}
(\sqrt{3}\pi i)^2 G(\xi_3 \eta_3; \xi_3' \eta_3') &= (\sqrt{3}\pi i)^2 G(\xi\eta; \xi'\eta') \\
&= \sum_{123} \Delta_1{}^{-1} \Delta_2{}^{-1} (\pi i \sqrt{3}) \int d\hat{q}_{12}'' M d\sigma_{12}'' K(\hat{q}_{12}, \hat{q}_{12}'') G(\xi_3'' \eta_3''; \xi_3' \eta_3') \\
&= \sum_{123} D(\hat{q}_{12}) \Delta_1{}^{-1} \Delta_2{}^{-1} \tilde{G}_{3\xi}(\hat{\xi}_3 \eta_3; \hat{\xi}_3' \eta_3') \Delta_1'{}^{-1} \Delta_2'{}^{-1} \qquad (B.4.2)
\end{aligned}$$

where the second form exploits the symmetry between ξ, η and ξ', η'.

At this stage, unlike the 2-body case, the reconstruction of the 4D Green's function is *not yet* complete for the 3-body case, as eq.(B.4.2) clearly shows. This is due to the *truncation* of Hilbert space implied in the ansatz of 3D support to the pairwise BSE kernel K which, while facilitating a 4D to 3D BSE reduction without extra charge, does *not* have the *complete* information to permit the *reverse* transition (3D to 4D) without additional assumptions; see [29] for details. The physical reasons for the 3D ansatz for the BSE kernel have been discussed in detail elsewhere [23,29], vis-a-vis contemporary approaches. Here we look upon this "inverse" problem as a purely *mathematical* one.

We must now look for a suitable ansatz for the quantity $\tilde{G}_{3\xi}$ on the right hand side of (B.4.2) in terms of *known* quantities, so that the reconstructed 4D G-function satisfies the 3D equation (B.3.12) exactly, as a "check-point" for the entire exercise. We therefore seek a structure of the form

$$\tilde{G}_{3\xi}(\hat{\xi}_3 \eta_3; \hat{\xi}_3' \eta_3') = \hat{G}(\hat{\xi}_3 \hat{\eta}_3; \hat{\xi}_3' \hat{\eta}_3') \times F(p_3, p_3') \qquad (B.4.3)$$

21

where the unknown function F must involve only the momentum of the spectator quark #3. A part of the $\eta_3, \eta_3{}'$ dependence has been absorbed in the $\hat{G}$ function on the right, so as to satisfy the requirements of S_3-symmetry for this 3D quantity [29].

As to the remaining factor F, it is necessary to choose its form in a careful manner so as to conform to the conservation of 4-momentum for the *free* propagation of the spectator between two neighbouring vertices, consistently with the symmetry between p_3 and $p_3{}'$. A possible choice consistent with these conditions is the form (see [29] for details):

$$F(p_3, p_3{}') = C_3 \Delta_3{}^{-1} \delta(\nu_3 - \nu_3{}') \tag{B.4.4}$$

Here $\Delta_3{}^{-1}$ represents the "free" propagation of quark #3 between successive vertices, while C_3 represents some residual effects which may at most depend on the 3-momentum $\hat{p}_3$, but must satisfy the main constraint that the 3D BSE, (B.3.12), be *explicitly* satisfied.

To check the self-consistency of the ansatz (B.4.4), integrate both sides of (B.4.2) w.r.t. $ds_3 ds_3{}' dt_3 dt_3{}'$ to recover the 3D S_3-invariant $\hat{G}$-function on the left hand side. Next, in the first form on the right hand side, integrate w.r.t. $ds_3 ds_3{}'$ on the G-function which alone involves these variables. This yields the quantity $\tilde{G}_{3\xi}$. At this stage, employ the ansatz (B.4.4) to integrate over $dt_3 dt_3{}'$. Consistency with the 3D BSE, eq.(B.3.12), now demands

$$C_3 \int \int d\nu_3 d\nu_3{}' \Delta_3{}^{-1} \delta(\nu_3 - \nu_3{}') = 1; (since dt = d\nu) \tag{B.4.5}$$

The 1D integration w.r.t. $d\nu_3$ may be evaluated as a contour integral over the propagator Δ^{-1}, which gives the pole at $\nu_3 = \hat{\omega}_3/M$, (see below for its definition). Evaluating the residue then gives

$$C_3 = i\pi/(M\hat{\omega}_3); \quad \hat{\omega}_3^2 = m_q{}^2 + \hat{p}_3^2 \tag{B.4.6}$$

which will reproduce the 3D BSE, eq.(B.3.12), *exactly*! Substitution of (B.4.4) in the second form of (B.4.2) finally gives the desired 3-body generalization of (B.2.12) in the form

$$3G(\xi\eta; \xi'\eta') = \sum_{123} D(\hat{q}_{12}) \Delta_{1F} \Delta_{2F} D(\hat{q}'_{12}) \Delta_{1F}{}' \Delta_{2F}{}' \hat{G}(\hat{\xi}_3\hat{\eta}_3; \hat{\xi}'_3\hat{\eta}'_3)[\Delta_{3F}/(M\pi\hat{\omega}_3)] \tag{B.4.7}$$

where for each index, $\Delta_F = -i\Delta^{-1}$ is the Feynman propagator.

To find the effect of the ansatz (B.4.4) on the 4D BS *wave function* $\Phi(\xi\eta; P)$, we do a spectral reduction like (B.3.13) for the 4D Green's function G on the left hand side of (B.4.2). Equating the residues on both sides gives the desired 4D-3D connection between Φ and ϕ:

$$\Phi(\xi\eta; P) = \sum_{123} D(\hat{q}_{12}) \Delta_1{}^{-1} \Delta_2{}^{-1} \phi(\hat{\xi}\hat{\eta}; P) \times \sqrt{\frac{\delta(\nu_3 - \hat{\omega}_3/M)}{M\hat{\omega}_3\Delta_3}} \tag{B.4.8}$$

¿From (B.4.8) and eq.(2.1) of the text, we infer the structure of the baryon-qqq vertex function V_3 as given in eq.(2.5) of the text. For a detailed discussion of the significance of this result, vis-a-vis contemporary approaches, see [29].

Appendix C: Gauge Corrections to Kaon E.M. Mass

We outline here a practical procedure to evaluate the gauge corrections to the e.m. self-energy of a $q\bar{q}$ system, vide fig.1b of [28], pending a more systematic treatment in a

later paper. This two-body exercise should hopefully serve as a fascimile of the effect expected for the present qqq case. For brevity we shall refer to the figures of KL [32] in their notation without drawing them anew. Thus fig 1b of [28] corresponds to fig 1a of KL [32], except for the presence of the hadron lines at the two ends. We shall call this simply '1a', with the understanding that the hadron lines are 'attached' to 1a. For the actual mathematical symbols (including phase conventions) we shall draw freely from [28], without explanation. In [28], only 1a of [32] was calculated, but now one must add 2(a,b,c,d,e) of [32], all with hadron lines understood at the two ends of each. There is no need to calculate 1b or 1c of [32] which are mere e.m. self-energies of single quarks (g.i. by themselves), and are routinely absorbed in quark mass renormalization (of little significance in this phenomenological study which has these masses as inputs).

A new ingredient is a 4-point vertex in each of 2(a,b,c,d), and *two* 4-point vertices in 2e, except that the word 'point' is now understood as an extended structure characterized by the hadron-quark vertex function $D(\hat{q})\phi(\hat{q})$ where one must insert a photon line in each such $Hq\bar{q}$ blob. Since it is *not* a standard point vertex, the method [32] of inserting exponential phase integrals with each current is not technically feasible; instead we may resort to the simple-minded substition $p_i - e_i A(x_i)$ for each 4-momentum p_i (in a mixed p, x representation) occurring in the structure of the vertex function, which has the same physical content, at least up to first order in the e.m. field, without further comment. This amounts to replacing each $\hat{q}_\mu$ occurring in $\Gamma(\hat{q}) = D(\hat{q})\phi(\hat{q})$, by $\hat{q}_\mu - e_q A_\mu$, where $e_q = \hat{m}_2 e_1 - \hat{m}_1 e_2$. The net result in the first order in A_μ is a first order correction to $\Gamma(\hat{q})$ of amount $e_q j(\hat{q}).A$ defined by

$$j(\hat{q}).A = -4M\hat{q}.A\phi(\hat{q})(1 - D(\hat{q})/(4M\beta^2)) \tag{C.1}$$

where we have made free use of various symbols and definitions in [28]. (The effect of the hat structure of $\hat{q}$ on the e.m. substitution is ignored in this approximate treatment). This effective 4-point vertex function is operative at one end in each of 2a,2b,2c,2d of KL [32] and at both ends of 2e. For the e.m. vertex at the quark lines of 2(a,b,c,d), we use simply $ie_i\gamma.A$, as in [28]. The matrix elements can now be written down on exactly the same lines, and the *same* phase convention as in [28] to keep proper track of the gauge corrections with sign. We need write these down only for 2a and 2e, noting the equalities 2a=2b, as also 2c=2d, and the further substitutions $(1) \rightarrow (2)$ and vice versa to generate 2c(=2d) from 2a(=2b). The contribution from 2a [32] to the e.m. quadratic self-energy of a kaon is expressible as

$$
\begin{aligned}
M_{2a}^2 &= N_H^2 (2\pi)^{-5} e_1 e_q \int j(\hat{q})_\mu D(\hat{q}')\phi(\hat{q}') Tr[\gamma_5 D_{F\mu\nu}(k) \\
&\quad S_F(p_1 - \hat{m}_1 k) ie_1 \gamma_\nu S_F(p_1')\gamma_5 S_F(-p_2')] d^4q d^4k
\end{aligned}
\tag{C.2}
$$

where $p_1' = p_1 + \hat{m}_2 k$ and $p_2 = p_2' = p_2 - \hat{m}_2 k$ are the 4-momenta of the quarks at the other (right-hand) end, and the photon propagator in the Landau gauge is $-i(\delta_{\mu\nu} - k_\mu k_\nu/k^2)/k^2$. To make better use of the techniques outlined in [28], it is convenient to change the variable from k_μ to q_μ', noting that $q' = q + \hat{m}_2 k$, which gives $d^4k = d^4q'/\hat{m}_2^4$, etc. This shows that fig 2a(=2b), where the photon line ends on the heavier quark m_1, gives a bigger contribution than does fig.2c(=2d) which would give $\hat{m}_1^{-4}$ arising from the d^4k-measure. Evaluating the traces, and integrating over the poles of the two time-like momenta q_0 and q_0' gives for the sum of the contributions from 2a-2d to the quadratic mass difference

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between $\bar{K}^0$ and K^- as a product of two 3D quadratures after some simplifications with factorable approximations a la [28]:

$$\delta M^2_{2(a-d)} = \frac{6N_H^2 M\delta(e_1e_q)}{(2\pi)^3\hat{m}_2^3}\int d^3\hat{q}\int d^3\hat{q}'\frac{\phi\phi'}{\hat{q}\hat{q}'\omega_{1k}}[1-\frac{D(\hat{q})}{4M\beta^2}]$$
$$[(\hat{q}^2(2-4/\pi)-\hat{q}\hat{q}'/3)(M^2-\delta m^2+D(\hat{q}')\omega_1'^{-1}/2+D(\hat{q}')\omega_2'^{-1}/2)$$
$$+\frac{1}{3}\hat{m}_2\hat{q}\hat{q}'(D(\hat{q}')\omega_2'^{-1}/2+M^2-\delta m^2)]+[1\leftrightarrow 2] \tag{C.3}$$

Here $\delta(e_ie_q)$ is the $\bar{K}^0$ minus K^- difference between the indicated charge factors associated with line 'i', while $\omega_{1,2}'^2 = m_{1,2}^2+\hat{q}'^2$ and $\omega_{1k}^2 = m_1^2+(\hat{q}-\hat{m}_1\hat{k})^2$.

Next the contribution to δM^2 arising from fig 2e of KL [32] which involves the product of two vertex blobs like (C.1) is given by

$$\delta M^2_{2e} = iN_H^2(2\pi)^{-5}e_q^2\int d^4qd^4kD_{F\mu\nu}(k)j(\hat{q})_\mu j(\hat{q})_\nu Tr[\gamma_5S_F(p_1-\hat{m}_1k)\gamma_5S_F(-p_2+\hat{m}_2k)] \tag{C.4}$$

This integral is somewhat different in structure from (C.2) in as much as k_μ is fully decoupled from either wave function ϕ, ϕ', both of which have the same argument $\hat{q}$. This makes it possible to integrate first over d^4k as well as the time-like component q_0 of q_μ neither of which is involved in the vertex function. The relevant integral after tracing and rearranging has the form

$$F(\hat{q}) = 3(-i)^2\int d^4k\int dq_0k^{-2}(\delta_{\mu\nu}-k_\mu k_\nu/k^2)$$
$$[\hat{q}^2-q_0^2+m_1m_2-\hat{m}_1\hat{m}_2(P-k)^2]/(\Delta_1\Delta_2) \tag{C.5}$$

where $\Delta_i = m_i^2+(p_i-\hat{m}_ik)^2$. The integral which is entirely convergent works out after some standard manipulations involving Feynman techniques as well as differentiation under integral signs as

$$F(\hat{q}) = 6\pi^3[m_1m_2+\hat{q}^2+\Lambda][\sqrt{\Lambda}-\sqrt{\Lambda-\hat{m}_1\hat{m}_2M^2}]/(\hat{m}_1\hat{m}_2M)^2 \tag{C.6}$$

where $\Lambda = \hat{m}_1\hat{m}_2M^2+D(\hat{q})/2M$. And the final expression for (C.4) in terms of (C.6) is

$$\delta M^2_{2e} = N_H^2(2\pi)^{-5}\delta(e_q^2)\int d^3\hat{q}j(\hat{q})^2F(\hat{q}) \tag{C.7}$$

Further evaluation of (C.3) and (C.7) can be made a la [28] in a straightforward way. The key ingredients are

$$\delta e_1e_q = 0.236e^2; \quad \delta e_2e_q = 0.139e^2; \quad \delta e_q^2 = -0.0294e^2. \tag{C.8}$$

The break-up of the final results for the diagrams 2(a-e) after dividing the results of(C.3) and (C.7) by 2M, since $\delta M^2 = 2M\delta M$, is (in MeV):

$$\delta M_{2a+2b} = -0.6996; \quad \delta M_{2c+2d} = +0.1358; \quad \delta M_{2e} = -0.0481; \quad \delta M_{tot} = -0.612MeV. \tag{C.9}$$

All these corrections, which reinforce one another due to a complex interplay of signs, add up to a figure which increases the value -1.032 MeV due to fig 1(b) found in [28], to -1.644 MeV. A more comprehensive paper for the gauge corrections to fig 1(a) of [32] for the other mesons, as well as the qqq baryon, will be communicated separately. This estimate has been used, on a percentage basis, as a facsimile for the gauge correction expected for the e.m. self energy of the nucleon, in sec.5.3 of text.

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Figure Captions

Fig.1: Diagrams for BS normalization of Baryon-qqq vertex function. 1(a) shows quark #1 emitting a zero momentum photon ($k = 0$); its last qq interaction was with #2, while #3 is the spectator. 1(b) is the same diagram with the roles of #1 and #2 interchanged. 1(c) denotes schematically two more such pairs of diagrams obtained with cyclical permutations of the indices (123) in pairs. The 4-momenta on the quark lines are shown as used in the text.

Fig.2: Diagrams for the two-point interactions of the quark lines with the mass shift operator $-\delta m \tau_3^{(1)}/2$ in place of the photon in fig.1, but otherwise with identical topological correspondence of figs.2(a,b,c) to figs.1(a,b,c).

Fig.3: Diagrams for the e.m. self-energy of the uud (proton) configuration. 3(III) is shown in detail with full momentum markings as employed in the text, and corresponds to quark #3 as the spectator, while the quark lines #1 and #2 are joined by a transverse photon line. Similarly 3(I) and 3(II) correspond to #1 and #2 respectively as spectators in turn. Note that, unlike in fig.1 and fig.2, the interchange of #1 and #2 in fig.3(III) does not give a new configuration.

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